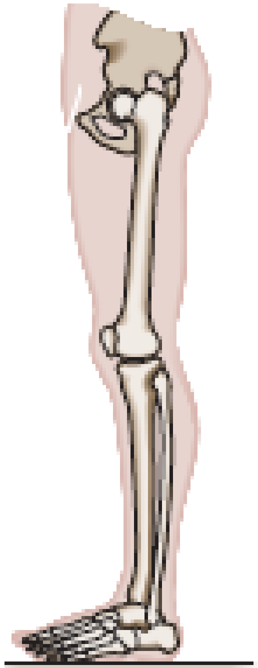
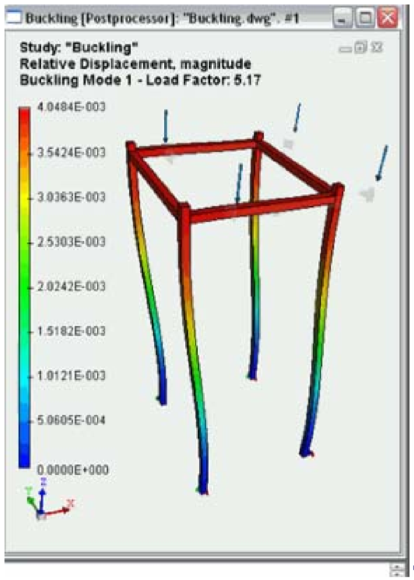
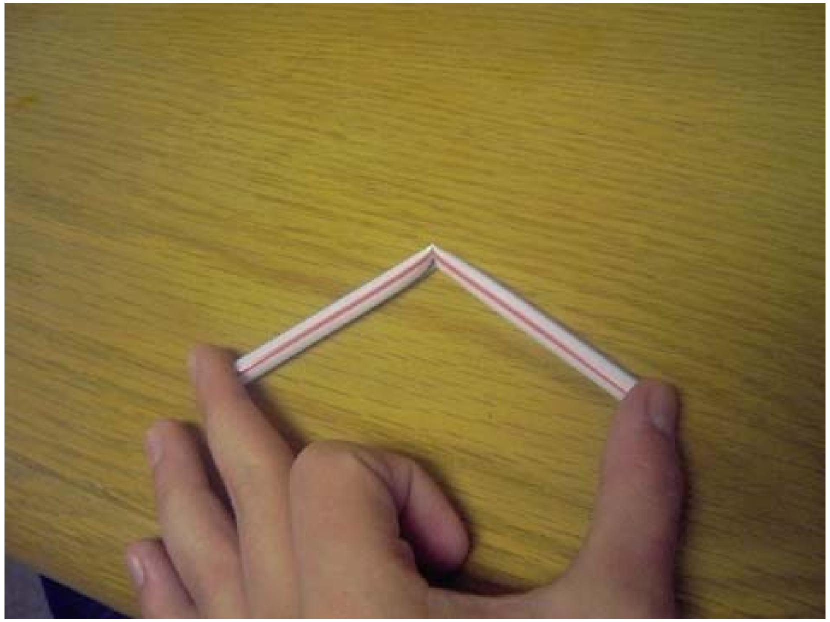
Izvijanje*





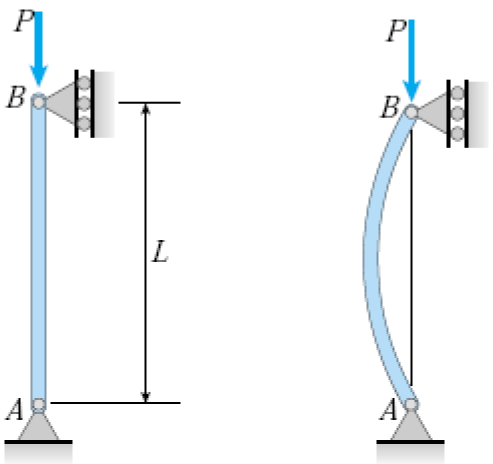


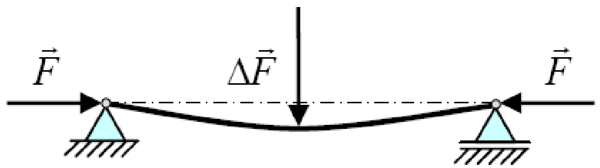
*JM Gere, BJ Goodno, *Mechanics of Materials*, Cengage Learning, Seventh Edition, 2009.
*Grupa autora, *Elastostatika II*, Tehnički fakultet, Bihać, 2003

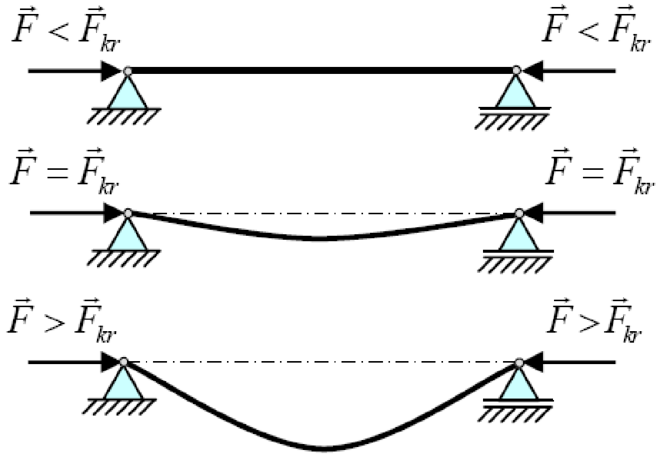
Osnovne karaktersitike i pojmovi

Stabilnost aksijalno pritisnutih elemenata

Umjesto kriterija čvrstoće (vrijednosti glavnih normalnih ili najvećih tangencijalnih napona ne prelaze kritične vrijednosti), ili kriterija krutosti (deformacije ne prelaze kritične veličine) kriterij koji se primijenjuje kod izvijanja je **kriterij stabilnosti**.





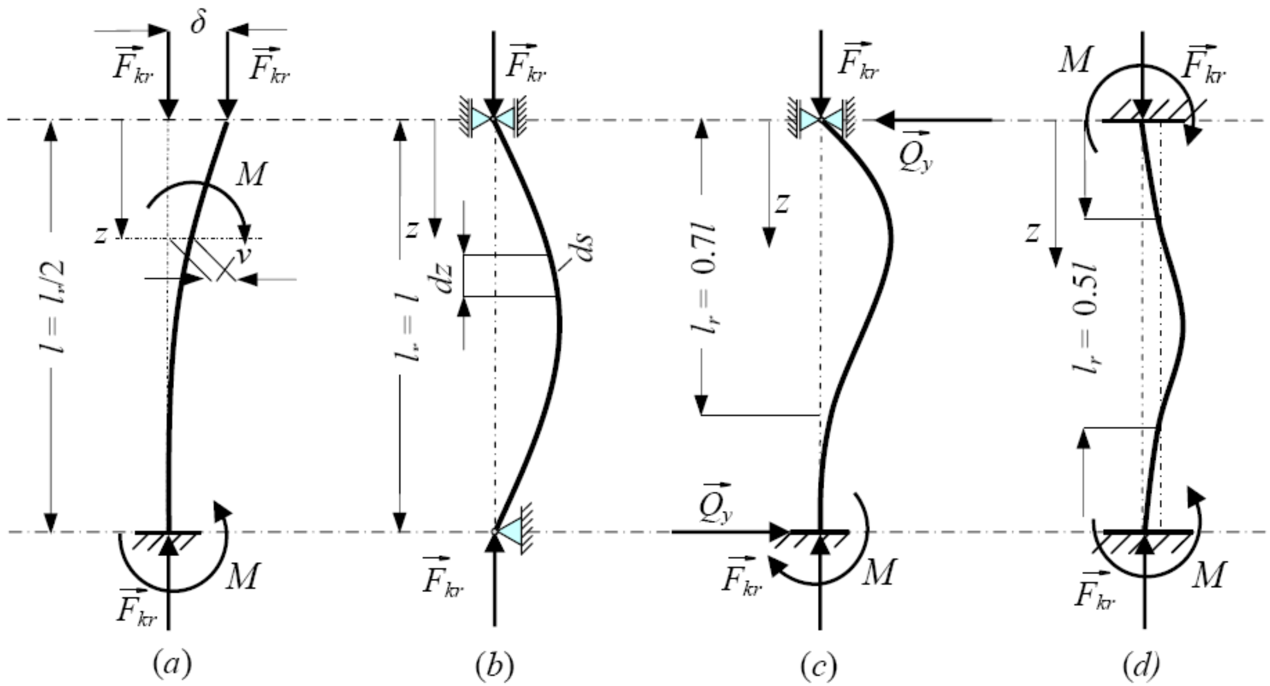


Euler-ova (Ojler) kritična sila izvijanja

Prelaz iz stabilnog u nestabilne uslove nastaje pri specifičnoj aksijalnoj sili, F_{kr} – **kritična sila.**

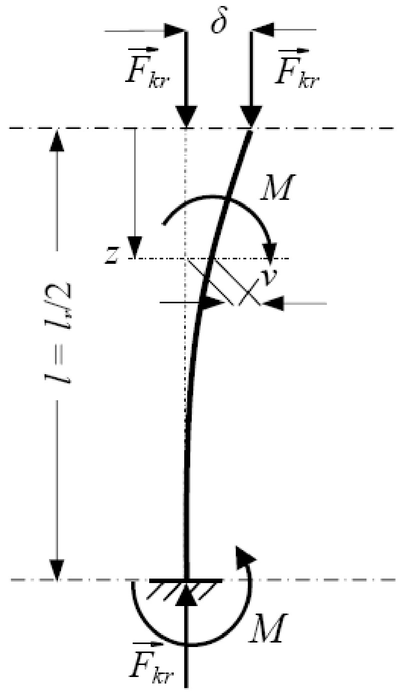
Za određivanje kritične sile koristi se diferencijalna jednačina elastične linije grede.

$$EI\frac{d^2v}{dz^2} = -M$$



Euler-ova (Ojler) kritična sila izvijanja

a) Konzola



(a)

$$EI\frac{d^2v}{dz^2} = -M$$

$$EI\frac{d^2v}{dz^2} = F(\delta - v) \qquad k = \sqrt{\frac{F}{EI}}$$

$$\frac{d^2v}{dz^2} + k^2v = k^2\delta \qquad \Rightarrow \qquad v = \delta + A\cos(kz) + B\sin(kz)$$

$$v(0) = \delta \qquad \Rightarrow \qquad A = 0$$

$$v'(L) = 0, \qquad v(L) = 0$$

$$B\cos(kl) = 0 \qquad B\sin(kl) = -\delta \qquad \Rightarrow \qquad v = \delta\left(1 - \frac{\sin(kz)}{\sin(kl)}\right)$$

$$kl = (2n-1)\frac{\pi}{2} \qquad n = 1, 2, 3\ldots$$

$$v = \delta\left\{1 + (-1)^n \sin\left[(2n-1)\frac{\pi}{2}\frac{z}{l}\right]\right\} \qquad n = 1, 2, 3\ldots$$

Osnovna forma izvijanja

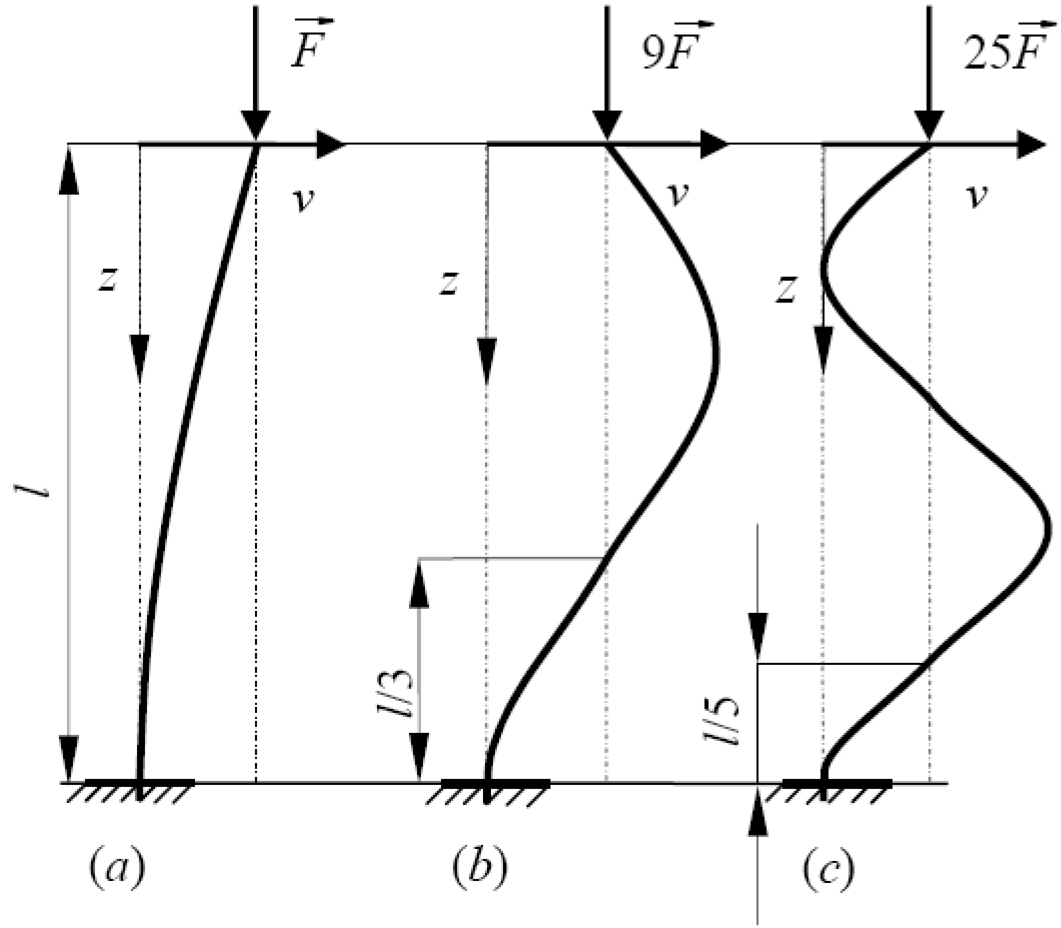
$$n = 1 \qquad \Rightarrow \qquad kl = l\sqrt{\frac{F_{kr}}{EI}} = \frac{\pi}{2}$$

$$F_{kr} = \frac{\pi^2 EI}{(2l)^2} \qquad I = I_{\min}$$

Euler-ova (Ojler) kritična sila izvijanja

a) Konzola – viši harmonici

$$F_{kr,n} = \left(2n-1\right)^2 \frac{\pi^2 EI}{\left(2l\right)^2}$$



Euler-ova (Ojler) kritična sila izvijanja

b) Prosta greda

$$EI\frac{d^2v}{dz^2} = -M$$

$$EI\frac{d^2v}{dz^2} = -Fv \qquad k = \sqrt{\frac{F}{EI}}$$

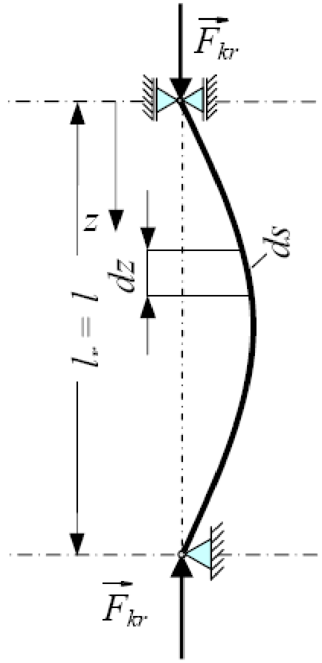
$$\frac{d^2v}{dz^2} + k^2v = 0 \quad \Rightarrow \quad v = A\cos(kz) + B\sin(kz)$$

$$v(0) = 0 \quad \Rightarrow \quad A = 0$$

$$v'(L) = 0 \quad \Rightarrow \quad B\sin(kl) = 0$$

$$kl = l\sqrt{\frac{F_{kr}}{EI}} = n\pi \qquad n = 1, 2, 3\ldots$$

$$F_{kr} = n^2\frac{\pi^2 EI}{l^2}$$



(b)

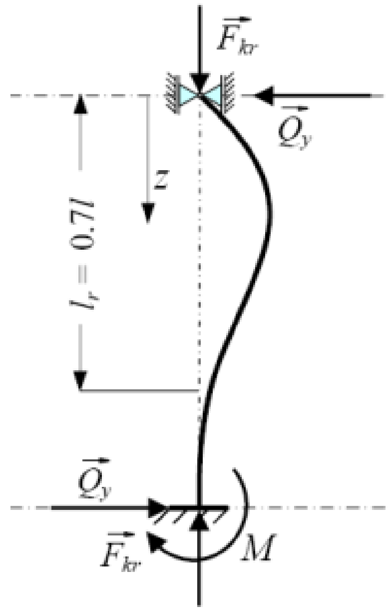
Osnovna forma izvijanja

$$n = 1$$

$$F_{kr} = F_e = \frac{\pi^2 EI}{l^2} \qquad I = I_{min}$$

Euler-ova (Ojler) kritična sila izvijanja

c) Greda s ukliještenjem



(c)

$$EI\frac{d^2v}{dz^2} = -M$$

$$EI\frac{d^2v}{dz^2} = -vF - zY \qquad k = \sqrt{\frac{F}{EI}}$$

$$\frac{d^2v}{dz^2} + k^2v = \frac{Y}{EI}z = \frac{Y}{F}k^2z \qquad \Rightarrow \qquad v = A\cos(kz) + B\sin(kz) + \frac{Y}{F}z$$

$$v(0) = 0 \qquad \Rightarrow \qquad A = 0$$

$$v(L) = 0 \qquad \Rightarrow \qquad B\sin(kl) = -\frac{Y}{F}l$$

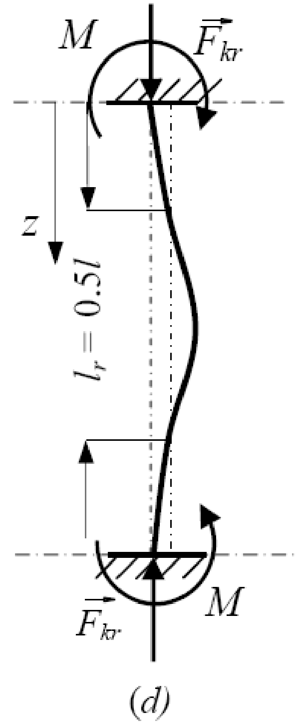
$$v'(L) = 0 \qquad \Rightarrow \qquad B\cos(kl) = -\frac{Y}{kF} \qquad \Rightarrow \qquad \mathrm{tg}(kl) = kl$$

Osnovna forma izvijanja

$$F_{kr} = (kl)^2\frac{EI}{l^2} = \left(\frac{kl}{\pi}\right)^2\frac{\pi^2EI}{l^2} = \frac{\pi^2EI}{\left[(\pi/kl)l\right]^2} \approx \frac{\pi^2EI}{(0.7l)^2}$$

Euler-ova (Ojler) kritična sila izvijanja

d) Greda s dva uklještenja



$$EI\frac{d^2v}{dz^2} = -M$$

$$EI\frac{d^2v}{dz^2} = -Fv + \mathrm{M} \qquad k = \sqrt{\frac{F}{EI}}$$

$$\frac{d^2v}{dz^2} + k^2 v = \frac{\mathrm{M}}{EI} \quad \Rightarrow \quad v = A\cos(kz) + B\sin(kz) + \frac{\mathrm{M}}{F}$$

$$v(0) = 0 \quad \Rightarrow \quad A = -\frac{\mathrm{M}}{F}$$

$$v'(0) = 0 \quad \Rightarrow \quad B = 0$$

$$v(L) = 0, \quad v'(L) = 0 \quad \Rightarrow \quad 1 - \cos(kl) = 0$$

$$kl = l\sqrt{\frac{F_{kr}}{EI}} = 2n\pi \qquad n = 1, 2, 3\ldots$$

$$F_{kr} = 4n^2\frac{\pi^2 EI}{l^2}$$

Osnovna forma izvijanja

$$n = 1$$

$$F_{kr} = F_e = \frac{\pi^2 EI}{(0.5l)^2} \qquad I = I_{\min}$$

Euler-ova (Ojler) kritična sila izvijanja – osnovna forma

a) Konzola

$$F_{kr} = \frac{\pi^2 EI}{(2l)^2} \qquad I = I_{\min}$$

b) Prosta greda

$$F_{kr} = \frac{\pi^2 EI}{l^2} \qquad I = I_{\min}$$

c) Greda s uklještenjem

$$F_{kr} = \frac{\pi^2 EI}{(0.7l)^2} \qquad I = I_{\min}$$

d) Greda s dva uklještenja

$$F_{kr} = \frac{\pi^2 EI}{(0.5l)^2} \qquad I = I_{\min}$$

l_r – redukovana dužina

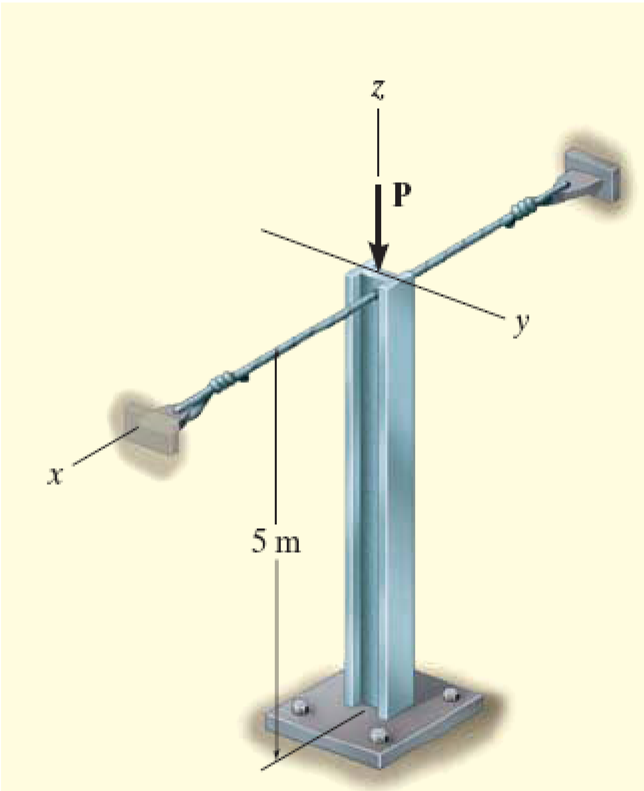
$$F_{kr} = \frac{\pi^2 EI}{l_r^2} \qquad I = I_{\min}$$

$$\sigma_{kr} = \frac{\pi^2 EI}{l_r^2 A} \qquad I = I_{\min}$$

Vitkost štapa λ – odnos redukovane dužine i minimalnog poluprečnika inercije

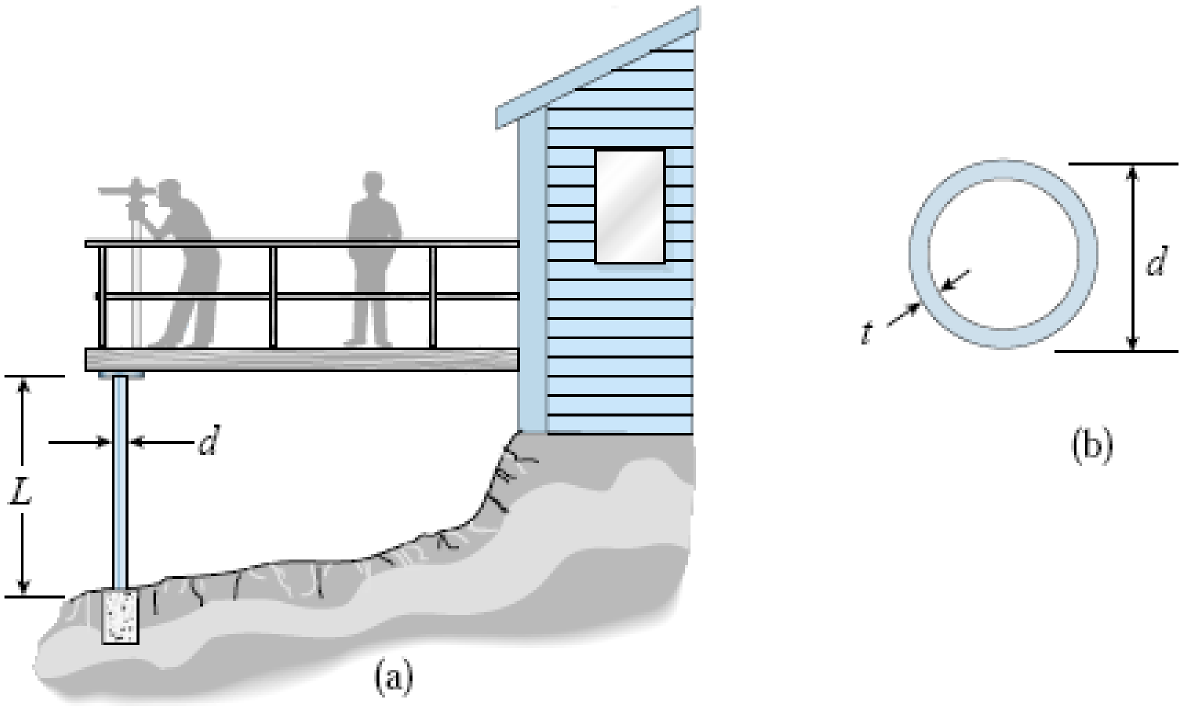
$$\lambda = \lambda_{kr} = \frac{l_r}{i_2} = \sqrt{\frac{A}{I_{\min}}} \qquad \Rightarrow \qquad \sigma_{kr} = \frac{\pi^2 E}{\lambda^2} = \frac{K}{\lambda^2}$$

Primjer 5.1: **Stub od aluminijuma je fiksiran na dnu te učvršćen na vrhu pomoću užadi kako bi se onemogućilo pomjeranje u pravcu *x*-ose, kao što je prikazano na slici. Odrediti najveću dozvoljenu silu *P* koja se može primijeniti, ako je faktor sigurnosti protiv izvijanja jednak 3. Uzeti da je E=70 GPa, R_{eH}=215 MPa, A=7500 mm^2, I_x=61.3·10^6 mm^4, I_y=23.2·10^6 mm^4.**



Primjer 5.2: **Platforma za osmatranje se oslanja nizom aluminijskih cijevi dužine 3.25 m, vanjskog prečnika 100 mm. Osnove cijevi su učvršćene u betonsku bazu, dok su gornji dijelovi spojeni s platformom. Cijevi su dizajnirane da izdrže opterećenje od 100 kN.**

Odrediti najmanju dozvoljenu debljinu cijevi ako je faktor sigurnosti jednak 3. Uzeti da je *E*=72 GPa, te granica proporcionalnosti 480 MPa.



(a) (b)

Ekscentrično izvijanje (formula sekante)

$$EI\frac{d^2v}{dx^2} = -M$$

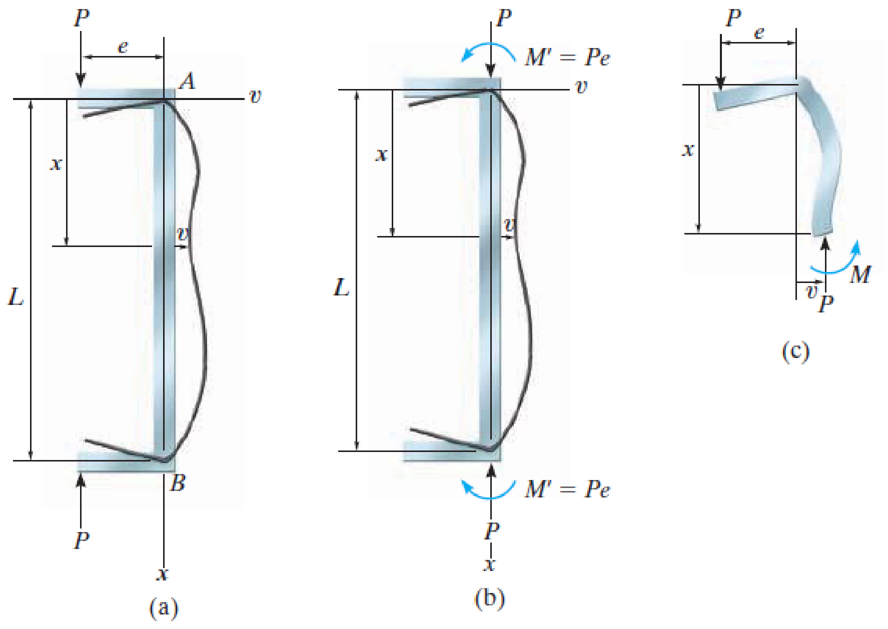
$$EI\frac{d^2v}{dx^2} = -P(e+v) \qquad k = \sqrt{\frac{P}{EI}}$$

$$\frac{d^2v}{dx^2} + k^2v = -k^2e \quad \Rightarrow \quad v = A\cos(kx) + B\sin(kx) - e$$

$$v(0) = 0 \quad \Rightarrow \quad A = e$$

$$v(L) = 0 \quad \Rightarrow \quad B = e\frac{1-\cos(kL)}{\sin(kL)} = e\tan\left(k\frac{L}{2}\right)$$

$$v = e\left[\cos(kx) + \tan\left(k\frac{L}{2}\right)\sin(kx) - 1\right]$$



Maksimalni ugib

$$v_{max} = e\left[\sec\left(k\frac{L}{2}\right) - 1\right]$$

Formula sekante

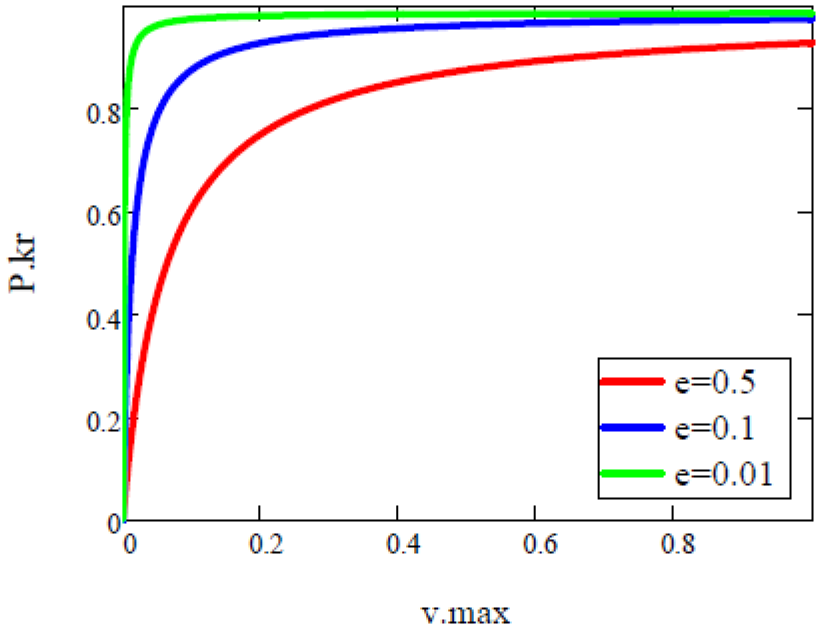
$$M = \left|P(e+v_{max})\right| = Pe\sec\left(k\frac{L}{2}\right)$$

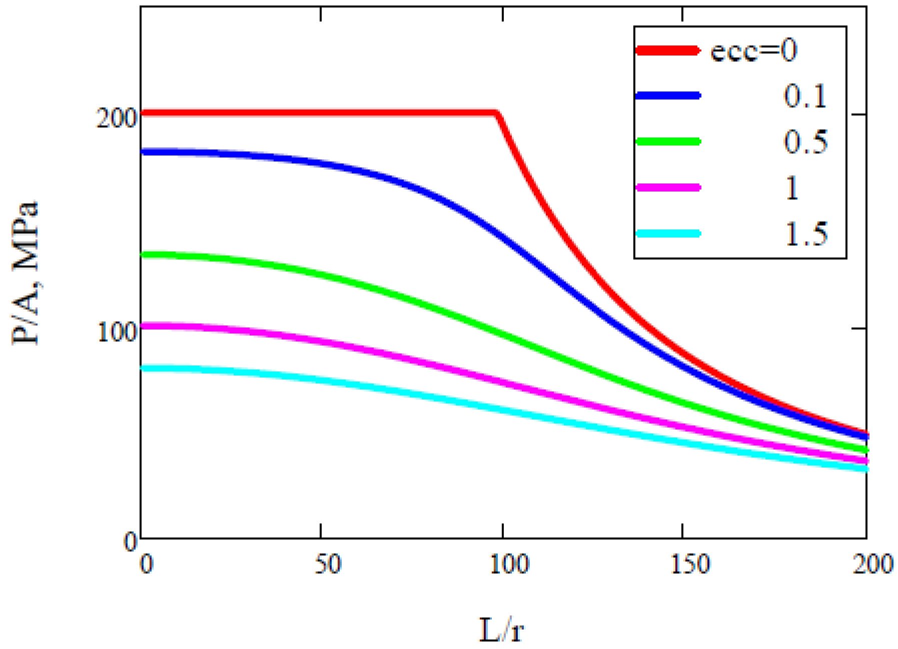
$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\sqrt{\frac{P}{EA}}\frac{L}{2r}\right)\right]$$

$$r = \sqrt{\frac{I}{A}}$$

Ekscentrično izvijanje (formula sekante)

$$v_{\max} = e\left[\sec\left(k\frac{L}{2}\right) - 1\right]$$





$R_{eH} = 200\text{MPa}$

$E = 200\text{GPa}$

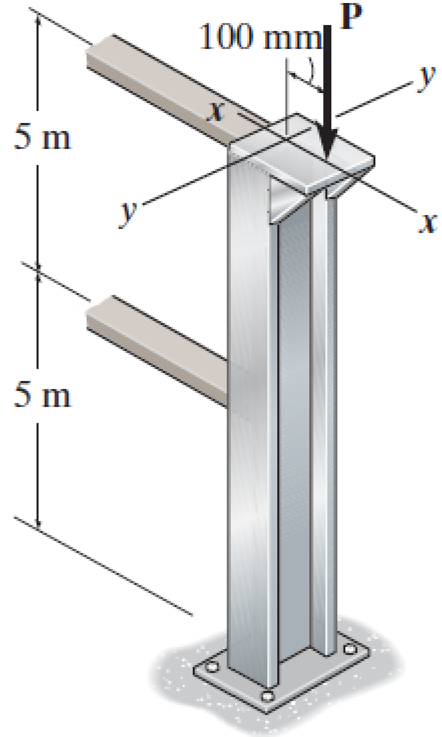
$$P_{kr}(v_{max}, ecc) = \frac{4\cdot E\cdot I\cdot \text{acos}\left(\frac{1}{v_{max}\cdot ecc^{-1} + 1}\right)^2}{L^2}$$

$$\sigma_{max}(ecc, x) = \frac{P}{A}\cdot\left(1 + ecc\cdot \sec\left(\sqrt{\frac{P}{E\cdot A}}\cdot\frac{x}{2}\right)\right) \qquad ecc = \frac{ec}{r^2} \qquad x = \frac{L}{r} \qquad r = \sqrt{\frac{I}{A}}$$

Ekscentrično izvijanje (formula sekante)

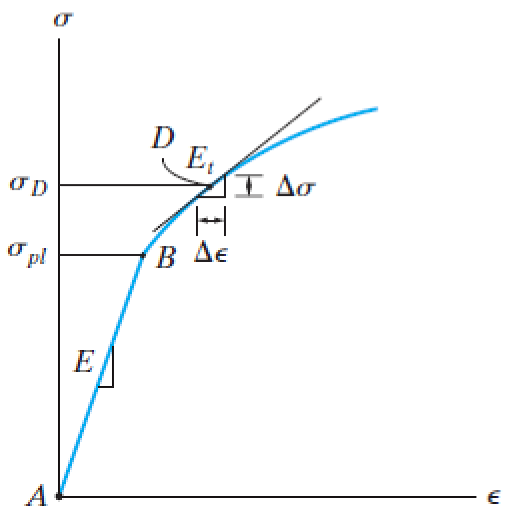
•13–53. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y–y* axis and free to move along the *y–y* axis. Also, the column is braced along the *x–x* axis at its mid-height. Determine the allowable eccentric force *P* that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.

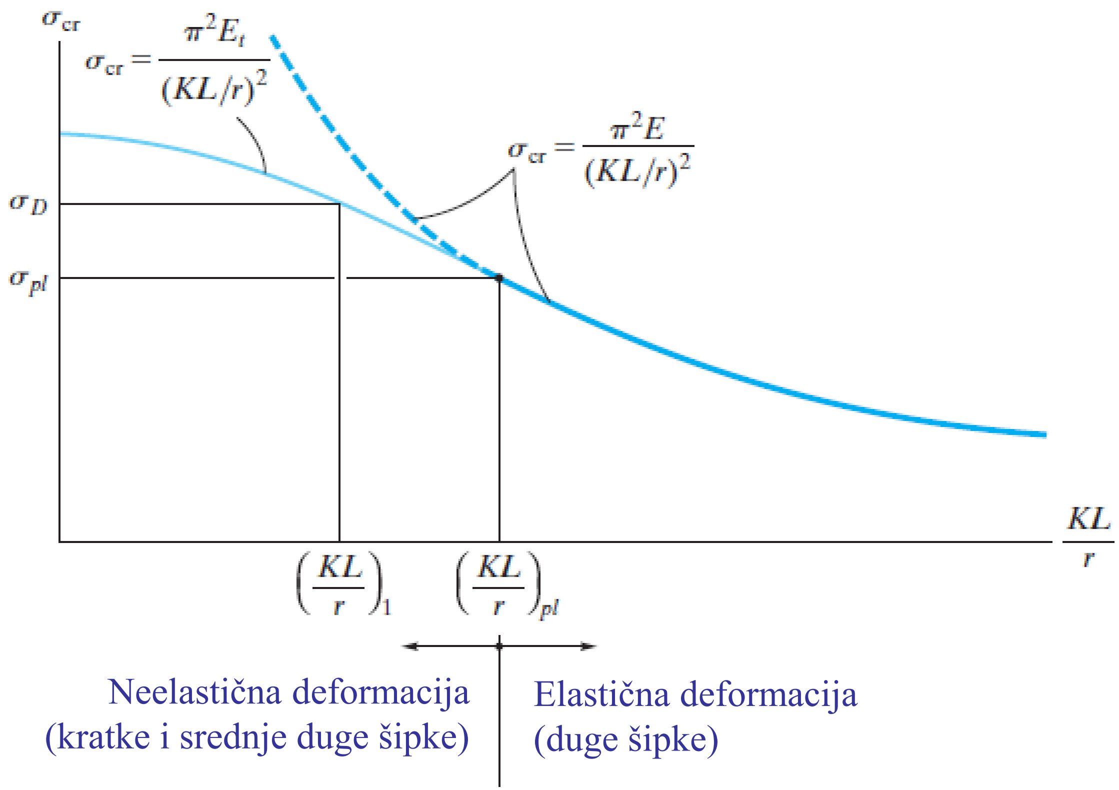
13–54. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y–y* axis and free to move along the *y–y* axis. Also, the column is braced along the *x–x* axis at its mid-height. If *P* = 25 kN, determine the maximum normal stress developed in the column.



Izvijanje u plastičnom području

Engesserov postupak





$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2}$$

Izvijanje u plastičnom području

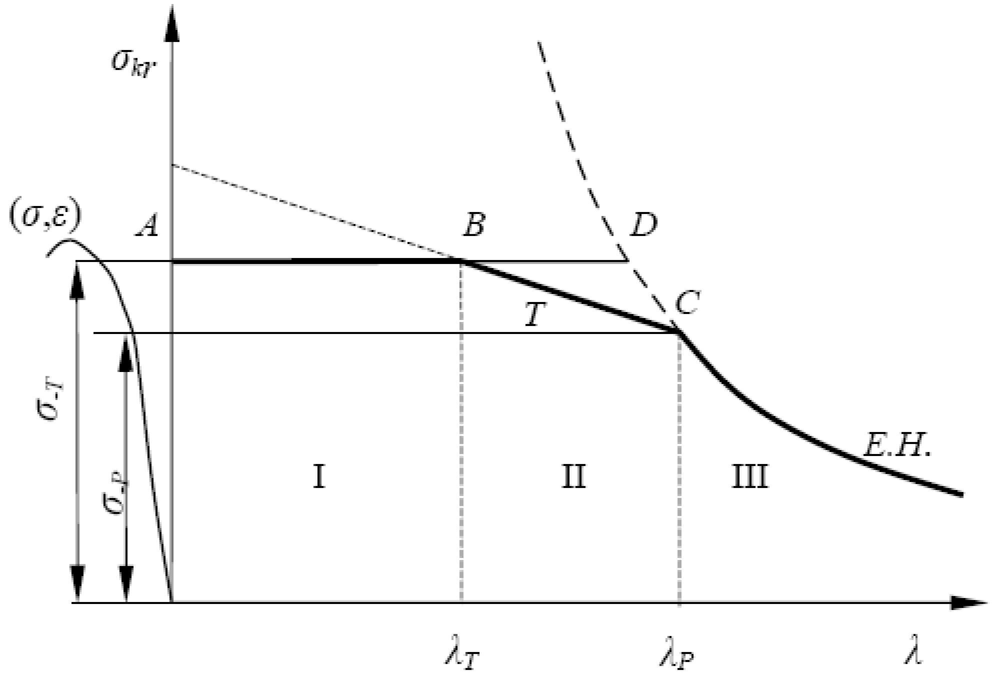
Ostali postupci ....

a) Empirijski obrasci (*Tetmeier, Ostenfeld - Johnson*)

b) Omega metoda

c) Energetska metoda

d) Ritz-ova metoda



13–75. The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.